UPSC Civil Services Main 1999 - Mathematics Algebra

Sunder Lal

Retired Professor of Mathematics

Panjab University

Chandigarh

December 16, 2007

Question 1(a) *If ϕ is a homomorphism of G into H with kernel K, then show that K is a normal subgroup of G.*

Solution. See Lemma 2.7.3 page 57 of Algebra by Herstein. ∎

Question 1(b) *If p is a prime number $p^\alpha \mid o(G)$, then prove that G has a subgroup of order p^α.*

Solution. This is Sylow's theorem. See theorem 2.12.1 page 92 of Algebra by Herstein. ∎

Question 1(c) *Let R be a commutative ring with unit element whose only ideals are the zero ideal and R itself. Show that R is a field.*

Solution. See Lemma 3.5.1 Page 137 of Algebra by Herstein. ∎