Sl. No. 0006832 A–LVV–O–UVD

STATISTICS—IV

Time Allowed : Three Hours	Maximum Marks : 200

INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions.

There are EIGHT questions divided under TWO Sections.

Candidate has to attempt FIVE questions in ALL.

Question Nos. 1 and 5 are compulsory and out of the remaining, THREE are to be attempted choosing at least ONE from each Section.

The number of marks carried by a question/part is indicated against it.

Candidates should attempt questions/parts as per the instructions given in the Section.

All parts and sub-parts of a question are to be attempted together in the answer-book.

Attempts of questions shall be counted in chronological order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the answer-book must be clearly struck off.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Answers must be written in ENGLISH only.

Section—I

1. *(a)* Illustrate the following terms with an example : 8

(i) Irreducible Markov chain

(ii) Renewal process

(iii) Wide-sense stationary process

(iv) Martingale

(b) If $N(t)$ is a Poisson process, then show that, as $t \to \infty$

$$P_r\left\{\left|\frac{N(t)}{t} - \lambda\right| > \varepsilon\right\} \to 0$$ 8

(c) Mention the situation(s) where 'periodic review policy' and 'continuous review policy' in inventory theory are used, and elaborate the ideas of the above policies. 8

(d) Prove that the number of basic variables in a balanced transportation problem is at most $m+n-1$, where m is the number of origins and n is the number of destinations. 8

(e) Give all basic solutions to the following :

$$12x_1 + 8x_2 + x_3 = 80$$
$$x_1 + 4x_2 + x_4 = 48$$

Are these solutions feasible solutions for an LPP setup? 8

2. *(a)* Define a Galton-Watson branching process. If $\{P_k\}$ is the offspring distribution, then prove the following identities : 10

(i) $P_n(s) = P_{n-1}[P(s)]$

(ii) $P_n(s) = P[P_{n-1}(s)]$

(b) *(i)* Discuss a one-dimensional random walk.

(ii) What is Gambler's ruin problem? Obtain the probability of ruin, if Gambler plays with a capital of a rupees. 10

(c) What is a Wiener process? Obtain the forward diffusion equation of a Wiener process. Also discuss any two applications of this process. 10

(d) Show that for $M/M/1/\infty$ queue model

$$P_n = \frac{\lambda_0 \lambda_1 \ldots \lambda_{n-1}}{\mu_1 \mu_2 \ldots \mu_n} P_0$$

where λ_n and μ_n are the means of Poisson and exponential distributions respectively when there are n people in the system. 10

3. *(a)* Solve the following problem by Simplex method : 10

Maximize $Z = 8x_1 + 6x_2$

subject to

$$x_1 + x_2 \leq 10$$
$$2x_1 + 3x_2 \leq 25$$
$$x_1 + 5x_2 \leq 35$$
$$x_1, x_2 \geq 0$$

(b) A solicitors' firm employs typists on hourly price-rate basis for their daily work. There are five typists and their charges and speeds are different. According to an earlier understanding, only one job is given to one typist and the typist is paid for a full hour even if he works for a fraction of an hour. Find the least cost allocation for the following data : 10

Typist	*Rate per hour (₹)*	*No. of pages typed/hour*
A	5	12
B	6	14
C	3	8
D	4	10
E	4	11

Job	*No. of pages*
P	199
Q	175
R	145
S	298
T	178

(c) Derive a formula to simulate a random observation from a Poisson random variable. 10

(d) What is the basis of classification of inventory in ABC analysis? Illustrate it with a simple example. 10

4. (a) Define a Poisson process. Stating the regularity conditions, show that $P_n(t) = P\{N(t) = n\}$ is given by the Poisson law

$$P_n(t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}, \quad n = 0, 1, 2, \ldots$$ 10

(b) If X_n is a branching process with

$$E(X_1) = m = \sum_{k=0}^{\infty} k p_k$$

and $\sigma^2 = \text{var}(X_1)$, then show that

(i) $E\{X_n\} = m^n$

(ii) $$\text{var}\{X_n\} = \begin{cases} \dfrac{m^{n-1}(m^n - 1)}{m-1}\sigma^2, & m \neq 1 \\ n\sigma^2, & m = 1 \end{cases}$$

10

(c) Trim Sheet Company requires rectangular sheets of width 5, 7 and 9 inches to be cut from a roll of width 20 inches. One-dimensional cutting is considered. The requirements are 300, 400 and 600 sheets of the three widths respectively. Write a linear programming problem to minimize waste assuming that the excess sheets cut are treated as waste. 10

(d) Given the following basic feasible solution to a transportation problem, obtain four iterations of the optimal solution using stepping stone method : 10

Demands

Sources					
	4	6	8	8	40
	20	20			
	6	8	6	7	60
			50	10	
	5	7	6	8	50
		10		40	
	20	30	50	50	

Section—II

5. Attempt *all* of the following : 8×5=40

(a) Explain stationary and stable population. State the situation when the age distribution of two stable populations would be identical. 8

(b) Compute the crude and standardized death rates of the populations *A* and *B*, regarding *A* as standard population, from the following data : 8

Age group (years)	*A*		*B*	
	Population	*Deaths*	*Population*	*Deaths*
Under 10	20000	600	12000	372
10–20	12000	240	30000	660
20–40	50000	1250	62000	1612
40–60	30000	1050	15000	525
Above 60	10000	500	3000	180

(c) Explain briefly the uses of life table. 8

(d) Explain stack and queue structures with suitable examples. Write the algorithm to 'add' and 'delete' into a stack and queue covering all possible cases. 8

(e) Write an algorithm/flowchart for computing product of two matrices A and B of orders $m \times n$ and $n \times p$ respectively. 8

6. (a) Discuss the main sources of demographic data in the Indian system. Give your suggestion for the measures to improve the quality of data. 10

(b) Compute GFR, SFR, TFR and GRR from the data given below :

Age group of child-bearing females	:	15–19	20–24	25–29	30–34
Number of women ('000)	:	16·0	16·4	15·8	15·2

Age group of child-bearing females	:	35–39	40–44	45–49
Number of women ('000)	:	14·8	15·0	14·5

Assume that the proportion of female births is 46·2 percent. 10

(c) Describe King's method for constructing an abridged life table. 10

(d) Explain the Matrix method of population projection. 10

7. *(a)* A list of n distinct integers is given. Write an algorithm/flowchart, using minimum number of comparison operations, to find out the second maximum number (i.e., the number less than the maximum number but greater than the others). 10

(b) Describe various types of keys, with appropriate examples, in regard to a database management system. 10

(c) Write down the algorithm to evaluate

$$\int_a^b f(x)\,dx$$

using Weddle's rule. How do you implement it in finding $P(X > 2)$, where $X \sim N(0, 1)$? 10

(d) Write an algorithm/flowchart to check whether an input non-zero integer n is prime. Prove that the main loop of the algorithm/flowchart for this purpose is to be executed up to $\lceil \sqrt{n} \rceil$ times, where $\lceil \sqrt{n} \rceil$ stands for the least integer greater than or equal to $\sqrt{n}$. 10

8. *(a)* Explain the meaning of the following rates in vital statistics and discuss their importance :

(i) Crude death rate

(ii) Standard birthrate

(iii) General fertility rate

Briefly indicate how these rates can be used in population projection. 10

(b) Describe the procedure and steps for the construction of life table. 10

(c) Describe various types of operating systems. 10

(d) Consider the following four numbers :

(1) $a = 567$ in octal representation

(2) $b = ABC$ in hexadecimal representation

(3) $c = 220$ in a hypothetical number system with base 3

(4) $d = 786$ in another hypothetical number system with base 9

Answer the following questions :

(i) Convert a directly into the corresponding number in hexadecimal representation.

(ii) Convert c directly into the corresponding number in the hypothetical number system with base 9.

(iii) Add a and b to produce the result in hexadecimal number system.

(iv) Add c and d to generate the result in the hypothetical number system with base 9. 10

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